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The problem of calculating the two-terminal reliability of a network having edges that fail randomly and independently is known to be NP-hard, even in the case of directed acyclic networks. This paper discusses an iterative technique that provides at each iteration both upper and lower bounds on the exact reliability value. These bounds are shown to converge to the exact answer for the case of acyclic networks. Computational results indicate that for certain classes of graphs these bounds converge rapidly and provide excellent approximations to the true network reliability.

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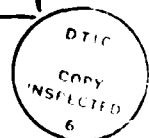
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ALGEBRAIC ASPECTS OF COMPUTING NETWORK RELIABILITY

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1. Introduction

It is important to be able to assess the reliability of a complex system, based on knowledge concerning the reliabilities of its individual components. Such problems arise with increasing frequency in the analysis of telecommunication, distribution and transportation systems, all of which can be modeled as networks of various sites (or nodes) interconnected by edges. In a typical situation, edges of the network (which may be directed or undirected) are assumed to fail in a statistically independent fashion with known probabilities. For such networks, a variety of probabilistic measures of system performance have been considered: namely, the probability that two given nodes can communicate [11,13,33], that a given node can communicate with a specified portion of the network [6,25], or that every pair of nodes can communicate [8,26,33].

Numerous algorithms have been proposed for calculating these measures of network reliability. One class of methods is based on the idea of a *path*, a minimal set of edges whose operation ensures that the system functions. In this approach, paths must first be enumerated and then combined either by applying the inclusion-exclusion principle or by effecting a partition into mutually disjoint events [2,13-15,17,27]. An alternative approach uses instead the enumeration and combination of *cutsets*, minimal sets of edges whose failure ensures that the system cannot function [1,11,12,17,22].

A different approach, called *pivotal decomposition*, considers the two states

(operative or failed) assumed by a given edge in the network [2,7,16,26]. Based on these two states, the given network decomposes into two smaller subnetworks. This process is repeatedly applied until all resulting subnetworks are solvable. Satyanarayana and Chang [26] have formulated and implemented an optimal edge selection strategy that achieves the minimum possible computational effort in the case of undirected networks.

Despite the considerable effort spent in developing algorithms for the calculation of network reliability, the effective computation of such probabilistic measures for general networks remains elusive. Namely, all known general procedures exhibit a worst-case behavior that is exponential in the size of the network. Since no polynomially-bounded algorithm has yet been devised, the exact computation of network reliability has been confined to networks of rather small size. In fact, most reliability problems of any substance are now known to be NP-hard or #P-complete [5,20-22,24,32], making it extremely unlikely that any polynomially-bounded algorithm will ever be found. As a result, recent research has concentrated on developing algorithms adapted to special network structures (where polynomial-time algorithms are possible), or has pursued a simulation approach [10].

A number of special classes of undirected networks have recently been analyzed with success. Polynomial-time algorithms are now available for calculating certain reliability measures in series-parallel [28], inner-four-cycle-free [18], and cube-free [19] planar graphs. Provan [20] has shown, however, that the problem of determining source-to-terminal reliability remains #P-complete for the general class of planar graphs. In order to analyze more complex networks topologies, the idea of pivotal decomposition together with polygon-to-chain reductions [34] can be used to decompose the original problem into smaller subproblems.

Similar results and tools are not as readily available in the case of directed networks. The only significant types of directed networks that are known to admit a polynomial-time algorithm are "basically-series-parallel" networks [3,4]. For general

directed networks, the algebraic approach developed by Shier and Whited [30,31] yields algorithms for solving, either approximately or exactly, some relatively complex source-to-terminal problems, although the computation time grows relentlessly with the size of the network.

This paper investigates a general algorithm for calculating source-to-terminal reliability in certain well-structured classes of networks, exemplified by planar graphs. The emphasis is on identifying an underlying lattice structure that captures certain algorithmically desirable features of such networks. The motivation for the present work derives from several recent results concerning cutset-based algorithms for network reliability. Specifically, Buzacott [9] discusses how reliability formulas using the disjoint products form can be simplified by making use of a certain ordering of cutsets in the network. Shanthikumar [29] has also exploited ordering properties of cutsets to obtain an efficient recursive algorithm to calculate an upper bound on source-to-terminal reliability. Most recently, Provan and Ball [22] have described a *pseudo-polynomial* algorithm for computing the reliability of an arbitrary source-to-terminal network (either directed or undirected). Namely, an algorithm is provided with worst-case complexity that is polynomial in the number of cutsets separating the source s and terminal t. Provan and Ball further show that for directed networks no algorithm polynomial in the number of (s,t) paths can exist unless P = NP.

For networks of reasonable size and sparsity, the collection of (s,t) paths can be quickly enumerated using a modified depth-first search of the network [23]. The generation of all (s,t) cutsets is not as straightforward, particularly in the case of directed networks. Thus, while a pseudo-polynomial algorithm exists relative to the number of cutsets, the effort required to generate such cutsets may be strikingly larger than that required to generate the corresponding paths. As an example, consider the dodecahedron graph given in Figure 1, having 20 nodes and 30 edges. This example has 7376 (s,t) cutsets but only 780 (s,t) paths. Thus, in this instance not only are the cutsets more difficult to generate than the paths, but there are substantially more

cutsets than paths involved.

The emphasis here is to crystallize those features of cutset-based algorithms that make them theoretically attractive (pseudo-polynomial behavior), while at the same time broadening the scope from cutsets to other, more easily computed, entities. We present a general framework and a general recursive algorithm defined on certain collections of objects (e.g., cutsets and paths) that admit a suitable partial ordering. This algorithm will run in time polynomial in the number of such objects.

2. Basic Definitions and Concepts

Let $E = \{e_1,...,e_n\}$ be a set of components, subject to failure, and let $X = \{X_1,...,X_r\}$ be a collection of subsets of E. Components in E have two states: *active* and *inactive*. A set $X_i \subseteq E$ is called *active* if all its components are active. We suppose that $(X,\geq)$ is a partial ordering having the lattice property: namely, any two X_i, $X_j \in X$ have a unique least upper bound $X_i \vee X_j$ and a unique greatest lower bound $X_i \wedge X_j$. There are two additional requirements imposed here.

<u>Closure</u>. If X_i is active and X_j is active, then $X_i \vee X_j$ and $X_i \wedge X_j$ are active. (1)

<u>Connectivity</u>. If $e \in X_i$ and $e \in X_j$ then $e \in [X_i, X_j] = \{ Z \in X: X_i \leq Z \leq X_j\}$. (2)

The event $\{X_i$ is active$\}$ will be denoted by A_i. We will be interested in calculating the quantity

$$\Omega(X) = Pr(A_1 \cup A_2 \cup ... \cup A_r).$$

<u>Example 1</u>. Consider the source-to-terminal undirected network G shown in Figure 2, with components (edges) $E = \{e_1,e_2,e_3,e_4,e_5\}$. Here the X_i's will be the (s,t) cutsets in the network defined by the following edge/node sets:

$$X_1 = \{e_1,e_2\} \qquad\qquad V_1 = \{s\}$$
$$X_2 = \{e_1,e_3,e_5\} \qquad\quad V_2 = \{s,b\}$$
$$X_3 = \{e_2,e_3,e_4\} \qquad\quad V_3 = \{s,a\}$$
$$X_4 = \{e_4,e_5\} \qquad\qquad V_4 = \{s,a,b\}$$

That is, the cutset X_i consists of all edges in $G = (V,E)$ joining nodes of V_i with nodes of $V-V_i$. The partial order $\geq$ is defined by

$$X_i \geq X_j \Leftrightarrow V_i \supseteq V_j .$$

The associated Hasse diagram for this partial order is shown in Figure 3. It is easy to

verify that requirements (1) and (2) both hold. By considering an "active" component to mean a "failed" component, the event A_i signifies that all components in X_i fail and $\Omega(X)$ is just the network *unreliability* $1\text{-}R_{st}(G)$: i.e., the probability that a message sent from s will be unable to reach t.

Example 2. Again consider the network in Figure 2, with the X_i's now denoting the (s,t) paths:

$$X_1 = \{e_2, e_5\}$$
$$X_2 = \{e_1, e_3, e_5\}$$
$$X_3 = \{e_2, e_3, e_4\}$$
$$X_4 = \{e_1, e_4\}$$

We define $X_i \geq X_j$ to mean that path X_i is geometrically "above" path X_j, yielding the same Hasse diagram shown in Figure 3. Again, requirements (1) and (2) are seen to hold. Here, the association of "active" with "functioning" allows A_i to be interpreted as the event that all components in X_i are functioning, whereupon $\Omega(X)$ is the source-to-terminal reliability $R_{st}(G)$.

As the above examples illustrate, we can define a partial ordering on the (s,t) cutsets in an arbitrary network or on the (s,t) paths in an (s,t)-planar network. In both cases, stipulations (1) and (2) are guaranteed hold. Other (non-network) examples of coherent systems [7] also exist which admit an ordering such that (1) and (2) are satisfied with respect to the system's "minpaths" or "mincuts". The following example illustrates one such case.

Example 3. Consider a system with components $\{e_1, e_2, \ldots, e_6\}$. The system functions if and only if all components function in at least one of the following subsets ("minpaths"):

$$X_1 = \{e_1, e_2, e_3\}$$

$$X_2 = \{e_2, e_3, e_4\}$$

$$X_3 = \{e_3, e_4, e_5\}$$

$$X_4 = \{e_4, e_5, e_6\}$$

If the (total) order $\geq$ is defined by $X_4 \geq X_3 \geq X_2 \geq X_1$ then requirements (1) and (2) will hold. In this case, the Hasse diagram for the partial order is simply a "chain," and the sets X_i do not correspond to the paths or cutsets of any network.

3. The Recursive Algorithm

In what follows, it will be assumed without loss of generality that the elements of the partial order have been *topologically ordered* : that is,

$$X_i < X_j \Rightarrow i < j . \tag{3}$$

We are interested in calculating $\Omega(X) = Pr(A)$, where $A = A_1 \cup A_2 \cup ... \cup A_r$. The events A_i are not disjoint, however. Accordingly, we define the events F_i (which will be disjoint) by

$$F_i = \{X_i \text{ is the "lowest" active set in } X\}.$$

These events are well defined in view of (1). Namely, if X_{i_1}, X_{i_2},..., X_{i_k} are active sets then so is $X_{i_1} \wedge X_{i_2} \wedge ... \wedge X_{i_k}$, and thus $X_{i_1} \wedge X_{i_2} \wedge ... \wedge X_{i_k} = X_{i_j}$ for some j. Notice that $X_{i_1} \wedge X_{i_2} \wedge ... \wedge X_{i_k} \leq X_{i_m}$ for all m, and so $X_{i_j} \leq X_{i_m}$ for all active sets X_{i_m}. Thus X_{i_j} is indeed the lowest active set in X.

Our general development will parallel that of Provan and Ball [22], which was originally stated in the specific context of (s,t) cutsets. First, it is easy to establish the following two properties of the F_i's.

Property 1. $F_i \cap F_j = \emptyset$ for $i \neq j$.

Proof: If F_i and F_j both occur for $i \neq j$ then X_i and X_j are active, so $X_i \wedge X_j$ is active by (1). Since $X_i \wedge X_j \leq X_i$ and $X_i \wedge X_j \leq X_j$ then $X_i = X_i \wedge X_j = X_j$, a contradiction. ♦

Property 2. $A = F_1 \cup F_2 \cup ... \cup F_r$.

Proof: Suppose A occurs with X_{i_1},..., X_{i_k} being active. Then $X_{i_1} \wedge ... \wedge X_{i_k} = X_{i_j}$ is active by (1) and so F_{i_j} occurs. Conversely, if F_j occurs then X_j is active and so A occurs. ♦

As a result of the above properties

$$\Omega(X) = Pr(A) = \sum_{j=1}^{r} Pr(F_j) \, , \tag{4}$$

and the problem reduces to that of calculating the $Pr(F_j)$. A general recursion involving these quantities can be easily derived as follows.

$$A_j = \bigcup_{i=1}^{r} (F_i \cap A_j)$$

$$= \left[\bigcup_{X_i < X_j} (F_i \cap A_j) \right] \cup F_j$$

$$= \left[\bigcup_{X_i < X_j} F_i \cap \{X_j - X_i \text{ is active}\} \right] \cup F_j \, . \tag{5}$$

The first equality follows from Property 2. The second follows since if $F_i \cap A_j$ occurs then $X_i \wedge X_j = X_i$, whence $X_i \leq X_j$. Because all unions in (5) involve disjoint events,

$$Pr(A_j) = \left[\sum_{X_i < X_j} Pr(F_i) \, Pr(X_j - X_i \text{ is active } | \, F_i) \right] + Pr(F_j) \, .$$

Now by requirement (2), $e \in X_j - X_i$ cannot be an element of any $X_k < X_i$. This means that the event F_i, which requires X_i to be active but all $X_k < X_i$ to be inactive, is independent of the event $\{X_j - X_i \text{ is active}\}$, whence

$$Pr(A_j) = \left[\sum_{X_i < X_j} Pr(F_i) \alpha_{ij} \right] + P(F_j) \, , \tag{6}$$

where

$$\alpha_{ij} = Pr(X_j - X_i \text{ is active})$$

$$= \prod_{e \in X_j - X_i} Pr(e \text{ is active}) \, ,$$

assuming the independence of component failures. Rearranging (6) gives the recursion (see [22])

$$Pr(F_j) = Pr(A_j) - \sum_{X_i < X_j} Pr(F_i) \, \alpha_{ij} \, . \tag{7}$$

The quantities A_i and α_{ij} are readily computed and thus $Pr(F_1)$, $Pr(F_2),..., Pr(F_r)$ can be found in turn using (7). The quantity $\Omega(X)$ can then be determined via (4). The worst-case complexity of the entire procedure is $0(m_X) \leq 0(|X|^2)$, where m_X indicates the number of arcs in the full partially ordered set $(X, \geq)$. Thus the above procedure is pseudo-polynomial: namely, its running time is bounded by a polynomial in the number of elements in the partial order.

Example 4. Consider the source-to-terminal network given in the upper portion of Figure 4. The ordering of paths from "top" to "bottom" gives the Hasse diagram shown in the lower portion of the figure. By using the shorthand notation $a_j \equiv Pr(A_j)$, $f_i \equiv Pr(F_i)$ and denoting the edge reliabilities as p_k, relation (7) yields:

$$f_1 = a_1 = p_2 p_7 .$$

$$f_2 = a_2 - f_1(p_5 p_6) = p_2 p_5 p_6 - p_2 p_5 p_6 p_7 .$$

$$f_3 = a_3 - f_1(p_1 p_3) = p_1 p_3 p_7 - p_1 p_2 p_3 p_7 .$$

and so forth, yielding $R_{st}(G) = \Omega(X) = \sum_{i=1}^{7} f_i .$

4. An Alternative Formulation

Equation (6), rewritten with our shorthand notation, becomes

$$a_j = \sum_{X_i \le X_j} f_i \alpha_{ij} \ , \quad \alpha_{jj} \equiv 1 \ .$$

We can now use Moebius inversion over the partially ordered set $(X, \ge)$ to obtain

$$f_j = \sum_{X_i \le X_j} a_i \mu_{ij} \ , \tag{8}$$

where μ_{ij} is the alternating sum of *chain values*, taken over all chains Q in $(X, \ge)$ joining X_i and X_j. That is,

$$\mu_{ij} = \begin{cases} 1 \ , & i = j \\ \sum_Q (-1)^{|Q|} \text{val}(Q) \ , & i \ne j \end{cases}$$

where $\text{val}(Q) = \prod_{e \in Q} \alpha_e$ and $|Q|$ denotes the number of arcs in chain Q. Algebraic simplification of (8) yields

$$f_j = \sum_{Q_j} (-1)^{|Q_j|} \sigma(Q_j) \ , \tag{9}$$

where the chains Q_j extend downward from X_j in $(X, \ge)$ and

$$\sigma(Q_j) = \prod \{ \text{Pr}(e) \colon e \in X_i \text{ for some } X_i \in Q_j \}$$

gives the joint probability of all edges appearing in some X_i along the chain. Analogously, when Q ranges over all chains in $(X, \ge)$ we obtain

$$\Omega(X) = \sum_Q (-1)^{|Q|} \sigma(Q) \ . \tag{10}$$

As an illustration, consider the bridge network in Figure 2 with the associated partially ordered set (based on paths) given in Figure 5. Then, using equation (9) yields, for example

$$f_4 = p_1 p_4 - (p_1 p_3 p_4 p_5 + p_1 p_2 p_4 p_5 + p_1 p_2 p_3 p_4) + 2p_1 p_2 p_3 p_4 p_5,$$

where first term corresponds to the chain of length 0, the next (negated) terms correspond to the three chains of length 1, and the final two terms correspond to the chains of length 2. The final expansion for $\Omega(X)$ resulting from (10) has 11 terms, corresponding to the 11 chains in $(X, \geq)$. This number is to be compared with the $2^4 - 1 = 15$ potentially occurring terms appearing in the inclusion/exclusion formula [2]. Thus equation (10) captures some of the cancellation found in the topological expansion formula of Satyanarayana and Prabhakar [27].

5. Computational Results

Promising computational results have been obtained using the recursive algorithm described in Section 3. We present in this section a fairly complete analysis of two particular networks (of moderate complexity) that have thus far resisted any thorough analysis.

The first network, with 14 nodes and 25 edges, is shown in Figure 6. This network (suggested by J. S. Provan) is known to be "self-dual," having 426 paths joining the specified s and t as well as 426 cutsets separating s and t. Since the network is (s,t)-planar, the paths can be ordered as described in Section 2 and the recursive algorithm can then be applied. In order to present the results succinctly, Table 1(a) presents the coefficients of the reliability polynomial when expressed as a function $R_{st}(p)$ of the common edge reliability $p_k = p$. Table 1(b) lists values of this polynomial at selected values of p throughout the range [0,1]. As expected, $R_{st}(0) = 0$, $R_{st}(1) = 1$, and $R_{st}(0.5) = 0.5$, with the latter value occurring because the network is self-dual. Computation of the relevant paths and the polynomial $R_{st}(p)$ required approximately 2.7 seconds using the IBM 3081-K computer at Clemson University.

A more complex network is the dodecahedron, shown in Figure 1. Since this network is not (s,t)-planar, the recursive formula was applied relative to the ordering produced using the (s,t) cutsets. Knowledge of the planarity of the network was, however, exploited in generating and processing the 7376 cutsets. Table 2(a) presents the coefficients of the unreliability polynomial $U_{st} = 1 - R_{st}$, now expressed for convenience as a function of the common edge failure probability $q_k = 1 - p_k = q$. Evaluation of $U_{st}(q)$ at selected values of q is given in Table 2(b). The total time required to generate the cutsets and produce $U_{st}(q)$ was under four and a half minutes of CPU time. It should be noted that the only previous calculations available for this example required several _hours_ of CPU time (on a similar mainframe) in order to obtain the value of $U_{st}(q)$ at two selected points (q = 0.1, q = 0.5).

Table 1. Reliability Polynomial $R_{st}(p)$ for Network in Figure 6

(a) Coefficients of $R_{st}(p)$:

$$R_{st}(p) = 4 p^4 + 18 p^5 + 30 p^6 - 38 p^7 - 188 p^8 - 154 p^9 + 128 p^{10} + 1456 p^{11}$$
$$+ 658 p^{12} - 3252 p^{13} - 4952 p^{14} - 1212 p^{15} + 37621 p^{16} - 1082 p^{17}$$
$$- 213712 p^{18} + 483698 p^{19} - 565880 p^{20} + 414334 p^{21} - 198122 p^{22}$$
$$+ 60628 p^{23} - 10850 p^{24} + 868 p^{25}$$

(b) Evaluation of $R_{st}(p)$ at selected points:

p :	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$R_{st}(p)$:	.0006	.0131	.0774	.2428	.5000	.7573	.9226	.9869	.9994

Table 2. Unreliability Polynomial $U_{st}(q)$ for Network in Figure 1

(a) Coefficients of $U_{st}(q)$:

$$U_{st}(q) = \; 2\,q^3 + 6\,q^4 + 18\,q^5 + 89\,q^6 + 216\,q^7 - 561\,q^8 - 5522\,q^9 - 4935\,q^{10}$$
$$+ 56046\,q^{11} + 133630\,q^{12} - 624126\,q^{13} - 1223316\,q^{14}$$
$$+ 8168604\,q^{15} - 4322481\,q^{16} - 63482286\,q^{17} + 256043573\,q^{18}$$
$$- 560270502\,q^{19} + 848614479\,q^{20} - 961097704q^{21} + 841480923\,q^{22}$$
$$- 577634706\,q^{23} + 311587945\,q^{24} - 131092176\,q^{25} + 42244830\,q^{26}$$
$$- 10090328\,q^{27} + 1685643\,q^{28} - 176016\,q^{29} + 8656\,q^{30}$$

(b) Evaluation of $U_{st}(q)$ at selected points:

q:	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$U_{st}(q)$:	.0029	.0362	.1685	.4272	.7097	.8975	.9767	.9973	.9999

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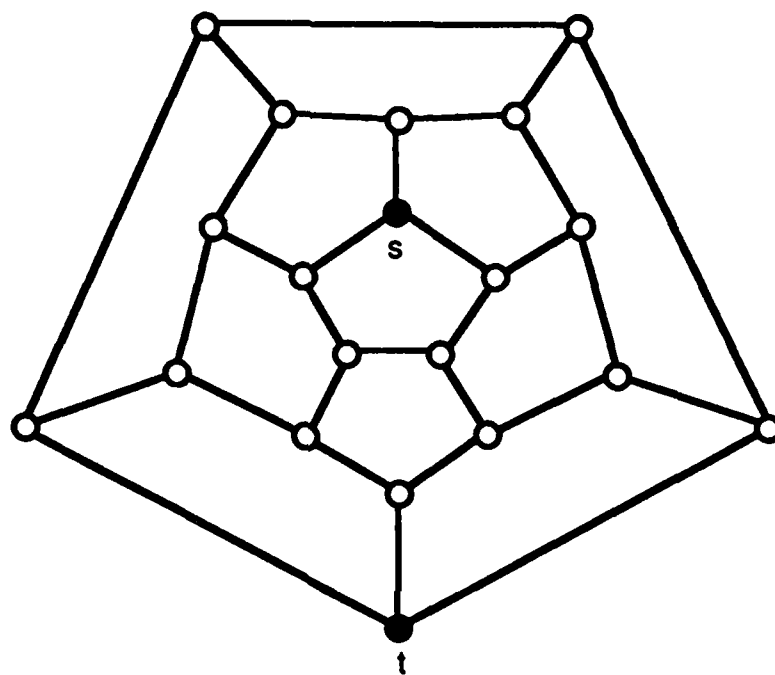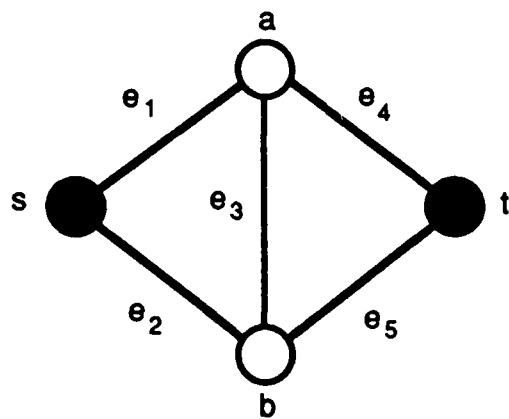[34] R. K. Wood, "A factoring algorithm using polygon-to-chain reductions for computing K-terminal network reliability," Networks, 15 (1985) 173-190.
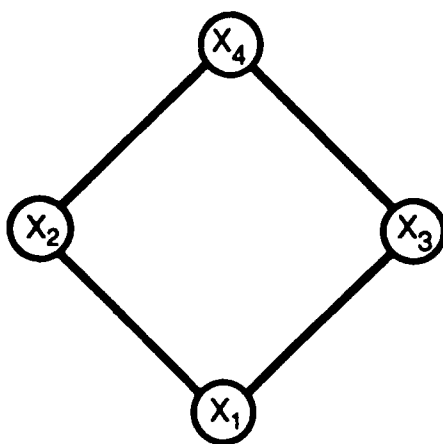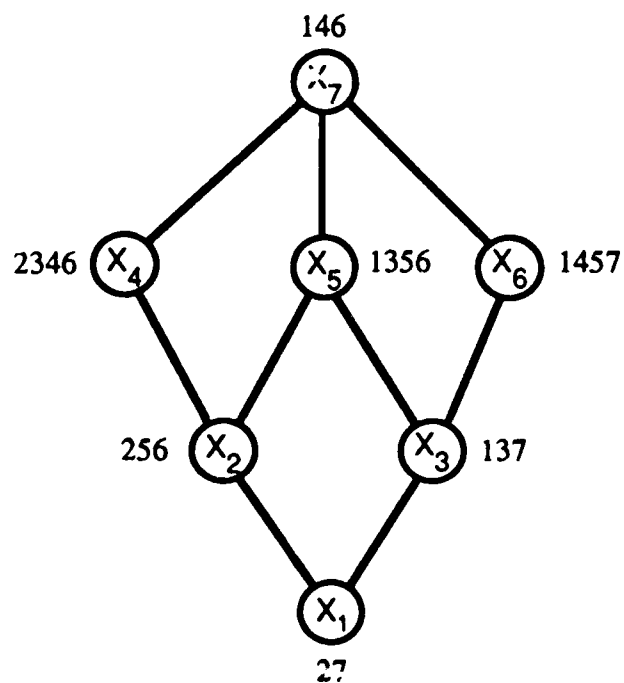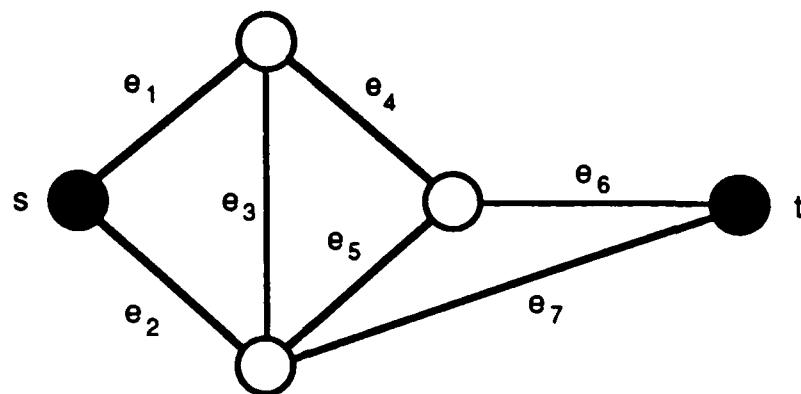
FIGURE 1

FIGURE 2

FIGURE 3

FIGURE 4
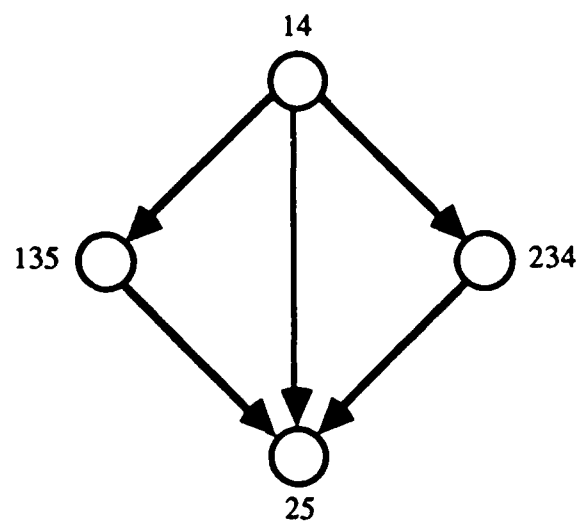
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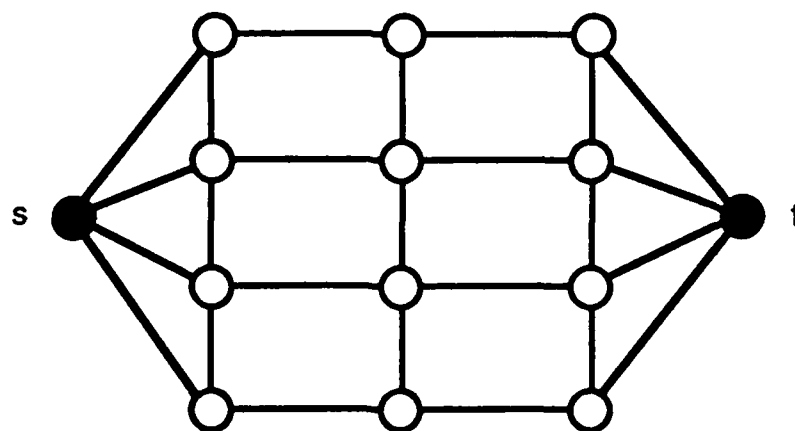
FIGURE 6

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